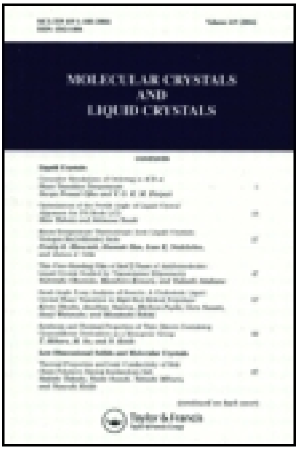

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The Stabilizing Effect of an Electric Field on the Shear Flow of Nematic Liquid Crystals When $\alpha_3>0$: Flow Alignment Regained

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The Stabilizing Effect of an Electric Field on the Shear Flow of Nematic Liquid Crystals When $\alpha_3 > 0$: Flow Alignment Regained

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Shear flow of a nematic liquid crystal with $\alpha_3 > 0$ is studied. It is shown that applying an electric field across the sample, a stable boundary layer type of flow can be obtained provided the field exceeds a critical value which is calculated. Expressions for the flow alignment angle, the boundary layer and the relaxation time are given as functions of the electric field and the stability of the director with respect to fluctuations which will take it out of the shear plane is discussed. By comparing the results to the well known corresponding expressions for shear flow when $\alpha_3 < 0$ and without an electric field applied, it can be seen that the two types of flow are very similar.

I INTRODUCTION

The application of a shear flow to a uniformly aligned nematic liquid crystal can lead to two entirely different situations, depending on the signs of the two Leslie viscosities α_2 and α_3.

a) $\alpha_2\alpha_3 > 0$: Assuming homeotropic boundary conditions, the simplest and best understood of the two possible cases is shown in Figure 1. In Figure 1a we show the situation for a very thin sample or for very small shear rates. The director will make an angle θ to the normal of the plates and θ will vary across the sample. The maximal tilt of the director, θ_m, will appear in the middle of the sample and it can be shown that θ_m can never exceed a limiting value θ_n which is the so called flow alignment angle. We will not further consider this type of "not fully developed" shear flow. If we increase the sample thickness or the shear rate (or both) we will very soon get the situation in Figure 1b. This

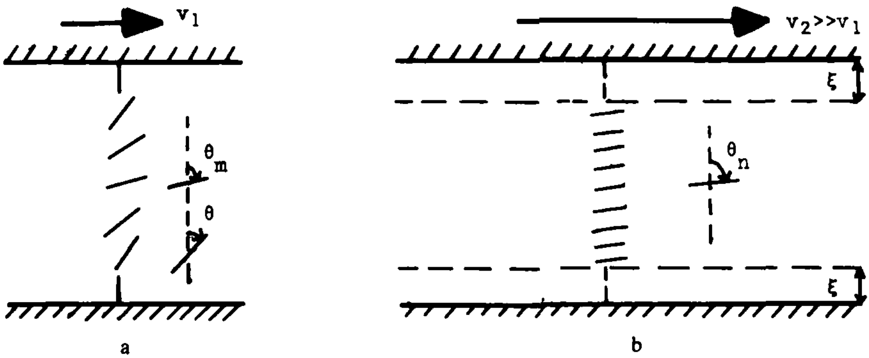

FIGURE 1 The director profile in shear flow when $\alpha_3 < 0$. (a) For small shears of a very thin sample the angle θ which the director makes with the normal will vary across the sample. (b) For larger shears we will get a boundary layer type of flow.

is a boundary layer type of flow: Except at two thin boundary layers of thickness ξ the director makes a constant angle θ_n to the normal of the plates. This stable type of shear flow is well described in the literature.[1,2]

b) $\alpha_2\alpha_3 < 0$: This type of flow which can occur is much less understood. Here it exists no flow alignment and a state commonly denoted tumbling will appear. The nature of this tumbled state is at present an open question. Some authors[3,4] describe tumbling as an unstationary state where the time derivative of the director is nonzero. Others[5,6] describe the situation as in Figure 2. Here we have a stationary flow but without flow alignment. The maximal tilt angle θ_m will increase without limit as the sample thickness increases. There are also indications that this last type of shear flow is unstable against rotations out of

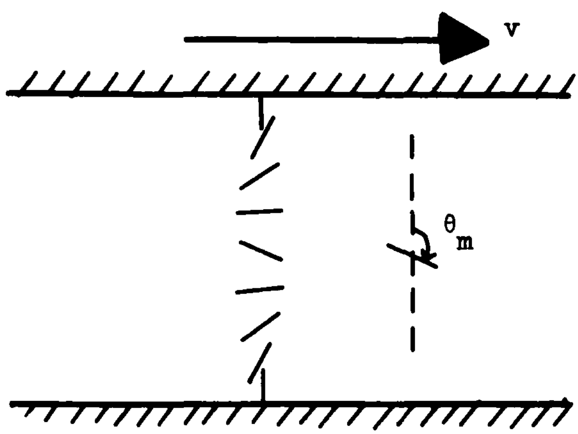

FIGURE 2 Possible director profile in shear flow when $\alpha_3 > 0$.

the shear plane so a two-dimensional description of the problem would be incomplete.[7]

To understand the difference in the flow behavior, let us study the viscous torque, Γ_y^v, exerted on the director. First we divide this torque into two parts.

$$\Gamma_y^v = \Gamma_y^d + \Gamma_y^h \tag{1}$$

The dissipative part, Γ_y^d, is a torque proportional to the angular velocity of the director, and will not be present in a stationary state. The other part, Γ_y^h, will be called the hydrodynamic torque and is due to the shear of the liquid crystal. In Figure 3 we define our coordinates. The lower plate is at rest while the upper one is moved in the x-direction with the velocity v_o. The z-direction is normal to the plates and the y-axis is pointing inwards. The angle which the director makes with the z-axis is denoted θ counting θ positive for a clockwise rotation. This means that a positive torque Γ_y will tend to increase θ while a negative Γ_y will tend to decrease it. The viscous torques are given by[2,8]

$$\Gamma_y^h = (\alpha_3 \sin^2 \theta - \alpha_2 \cos^2 \theta) u' \tag{2}$$

$$\Gamma_y^d = -\gamma_1 \frac{\partial \theta}{\partial t} \tag{3}$$

where u' is the local shear. The viscosity coefficient γ_1 can be shown[9] by thermodynamical reasoning to be always positive and is given by

$$\gamma_1 = \alpha_3 - \alpha_2 > 0 \tag{4}$$

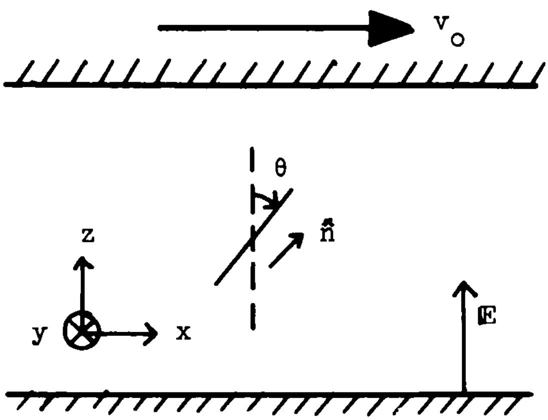

FIGURE 3 Definition of coordinates in the present work. The lower plate is at rest while the upper one is moved in the x-direction with the velocity v_o. The z-direction is normal to the plates and the y-axis is pointing inwards. The angle which the director makes with the z-axis is denoted θ counting θ positive for a clockwise rotation.

This guarantees that the dissipative torque will act as a frictional damping torque against a given rotation of the director.

Assuming now that our sample is thick enough so that we can neglect the influence of the walls, we study the stationary types of flow that can occur. Stationary means that all time derivatives equals zero so Γ_y^d will not matter in this discussion. The condition for the flow in Figure 1b to occur is that Γ_y^h is zero

$$\alpha_3 \sin^2 \theta - \alpha_2 \cos^2 \theta = 0 \tag{5}$$

This condition can be satisfied if α_2 and α_3 are of the same sign.

The case when both α_2 and α_3 are negative leads to a flow alignment angle θ_n given by

$$\tan \theta_n = (\overset{+}{-})\sqrt{\frac{\alpha_2}{\alpha_3}} \tag{6}$$

The minus sign in the solution is ruled out by stability reasoning.[10] The case when both α_2 and α_3 is positive would also give a stable flow but one has not so far found a nematic with positive α_2.

If α_2 and α_3 have different signs we have no solution to Eq. (5). The case when $\alpha_2 > 0$ and $\alpha_3 < 0$ is theoretically impossible due to Eq. (4). On the other hand there are today several nematics which fulfill the condition $\alpha_2 < 0$ and $\alpha_3 > 0$. Among those mentioned in the literature[3,4,11,12] we find HBAB (hexylo-amino-benzo-nitrile), CBOOA (cyano-benzylidine-octyloxy-aniline) and 8CB (octyl-cyano-biphenyl). As mentioned earlier these compounds show flow patterns which are not fully understood. However, in this work we will show that applying an electric field across the sample will stabilize the flow and for a large enough field we will regain the situation in Figure 1b.

We will proceed as follows. In Section II we will put up the general time dependent director equation including the effects of an electric field. In Section III we will deduce the well known expressions for the flow alignment angle θ_n, the boundary layer ξ and the relaxation time τ for the shear flow when $\alpha_3 < 0$ and without an electric field applied. In Section IV we will study the problem when $\alpha_3 > 0$ and with an electric field present. We will show that if the electric field exceeds a limiting value we will regain the stable flow in Figure 1. We will then calculate expressions for θ_n, ξ and τ as functions of the electric field. Comparing these with the results from Section III we can see that we have regained a boundary type of flow very similar to that with $\alpha_3 < 0$. In Section V we will discuss our results and also examine the validity of the approximations that we have made in our calculations.

II THE BALANCE OF TORQUE EQUATION

In Section I we gave the expression for the viscous torque per unit volume acting on the director. In order to put up the full equation of motion for the director we first have to write down expressions for the elastic and the electric torques present. In the one-constant approximation the elastic torque Γ_y^{el} can be shown to be[2]

$$\Gamma_y^{el} = K \frac{\partial^2 \theta}{\partial z^2} \tag{7}$$

where K is some kind of mean value of the two Frank elastic constants K_{11} and K_{33}. Applying an electric field $E = \hat{z}\sqrt{2}E_o \cos 2\pi\nu t$ will give rise to an electric torque[13] Γ_y^ϵ (Note however that we are using the SI-units)

$$\Gamma_y^\epsilon = -\tfrac{1}{2}\,\epsilon_a E_o^2 \sin 2\theta \tag{8}$$

where ϵ_a is the dielectric anisotropy of the nematic. We then get our equation for the director by equalizing the sum of all torques with $J\,\partial^2\theta/\partial t^2$, J being the moment of inertia per unit volume of the liquid crystal.

$$J \frac{\partial^2 \theta}{\partial t^2} = \Gamma_y^{el} + \Gamma_y^{v} + \Gamma_y^{\epsilon} \tag{9}$$

One can show[14] that the moment of inertia term is completely negligible in this problem and we end up with

$$-\gamma_1 \frac{\partial \theta}{\partial t} + K \frac{\partial^2 \theta}{\partial z^2} + (|\alpha_2|\cos^2\theta + \alpha_3 \sin^2\theta)u' - \frac{1}{2}\epsilon_a E_o^2 \sin 2\theta = 0 \tag{10}$$

As α_2 will be negative throughout the rest of this work, we have substituted α_2 with $-|\alpha_2|$ in Eq. (10). This will make it easier to see what happens in our equations as we let α_3 become both positive and negative.

We should also remind that both u' and E_o is far from constant in the general case but will be complicated functions of θ. Taking this into account changes the problem from solving one partial differential equation (Eq. 10) to solving a system of coupled partial differential equations. However in the boundary layer type of flow which we will study θ is constant over almost all the sample and it is a reasonable approximation to treat u' and E_o as constants, $u' = v_o/d$ and $E_o = U_o/d$ where U_o is the rms-value of the ac-voltage applied across the two plates.

III THE SHEAR FLOW WHEN $\alpha_3 < 0$

In this section we briefly discuss the shear flow without an electric field present and with $\alpha_3 < 0$. We assume a large enough shear rate to produce the flow pattern of Figure 1b. In order to easier be able to follow the calculations in Section IV, we will demonstrate how the expressions for θ_n, ξ and τ can be derived from Eq. (10).

Without the electric field term Eq. (10) will read

$$-\gamma_1 \frac{\partial \theta}{\partial t} + K \frac{\partial^2 \theta}{\partial z^2} + (\alpha_3 \sin^2 \theta - \alpha_2 \cos^2 \theta) u' = 0 \tag{11}$$

We notice that γ_1 is positive. The condition for finding the flow alignment angle is that $\partial\theta/\partial t$ and $\partial^2\theta/\partial z^2$ both equal zero. This is already given in Eq. (5) and the result for the flow alignment angle is stated in Eq. (6).

To calculate ξ and τ we proceed as follows. We expand Eq. (11) to linear order in $\delta\theta = \theta - \theta_n$. First we get

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial t}(\delta\theta) \tag{12}$$

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{\partial^2}{\partial z^2}(\delta\theta) \tag{13}$$

$$\sin^2 \theta \simeq \sin^2 \theta_n + 2 \sin \theta_n \cos \theta_n \delta\theta \tag{14}$$

$$\cos^2 \theta \simeq \cos^2 \theta_n - 2 \cos \theta_n \sin \theta_n \delta\theta \tag{15}$$

Substituting Eqs. (12)–(15) into Eq. (11) give

$$-\gamma_1 \frac{\partial}{\partial t}(\delta\theta) + K \frac{\partial^2}{\partial z^2}(\delta\theta) + (\alpha_3 \sin^2 \theta_n - \alpha_2 \cos^2 \theta_n) u' + \delta\theta u' (\alpha_3 + \alpha_2) \sin 2\theta_n = 0 \tag{16}$$

The third term in Eq. (16) equals zero according to Eq. (5). If we use the relationship

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \tag{17}$$

the expression for the flow alignment angle from Eq. (6) substituted into Eq. (17) gives

$$\sin 2\theta_n = -\frac{2\sqrt{\alpha_2 \alpha_3}}{\alpha_2 + \alpha_3} \tag{18}$$

Eq. (16) can now be rewritten

$$-\gamma_1 \frac{\partial}{\partial t}(\delta\theta) + K \frac{\partial^2}{\partial z^2}(\delta\theta) = Q \delta\theta \tag{19}$$

where Q is given by

$$Q = 2u'\sqrt{\alpha_2\alpha_3} \tag{20}$$

Studying a stationary flow we put $\partial\theta/\partial t \equiv 0$ in Eq. (19). A disturbance $\delta\theta_o$ of the director originating from the boundary conditions will then die out according to

$$K\frac{\partial^2}{\partial z^2}(\delta\theta) = Q\delta\theta \tag{21}$$

This equation has the solution

$$\delta\theta = \delta\theta_o e^{-z/\xi} \tag{22}$$

where ξ is our boundary layer given by

$$\xi = \sqrt{K/Q} = \sqrt{K/2u'\sqrt{\alpha_2\alpha_3}} \tag{23}$$

Assuming instead a homogeneous orientation in space we can also calculate the relaxation time for a nonequilibrium director orientation to reach flow alignment. We now put $\partial^2/\partial z^2 \equiv 0$ in Eq. (19) and get

$$-\gamma_1\frac{\partial}{\partial t}(\delta\theta) = Q\delta\theta \tag{24}$$

with the solution

$$\delta\theta = \delta\theta_o e^{-t/\tau} \tag{25}$$

where τ is the relaxation time given by

$$\tau = \gamma_1/Q = \gamma_1/2u'\sqrt{\alpha_2\alpha_3} \tag{26}$$

IV THE SHEAR FLOW WHEN $\alpha_3 > 0$ WITH APPLIED ELECTRIC FIELD

We shall now study the stabilizing effect of an electric field on the shear flow when α_3 is positive. Before going further we note that we restrict the director to be in the xz-plane, thus neglecting fluctuations that would take the director out of the shear plane. In the end of this section we then discuss the stability of our solutions with respect to these fluctuations showing that for $\epsilon_a > 0$ the solution is stable, but for $\epsilon_a < 0$ the solution is probably unstable for high enough fields and/or shear rates. However, of the substances known today with positive α_3 there is no one which has a negative ϵ_a, leaving this type of instability of no practical interest at present.

Equation (10) is the equation governing the flow and in order to investigate the possibility of flow alignment we put $\partial\theta/\partial t \equiv 0$ and $\partial^2\theta/\partial z^2 \equiv 0$. Using the relationship

$$\sin 2\theta = 2 \sin\theta \cos\theta \tag{27}$$

Eq. (10) will read

$$(|\alpha_2|\cos^2\theta + \alpha_3 \sin^2\theta)u' - \epsilon_a E_o \sin\theta \cos\theta = 0 \tag{28}$$

Dividing Eq. (28) with $\alpha_3 u' \cos^2\theta$ gives

$$\tan^2\theta - \frac{\epsilon_a E_o^2}{u'\alpha_3}\tan\theta + \frac{|\alpha_2|}{\alpha_3} = 0 \tag{29}$$

which has the solution

$$\tan\theta_n = \frac{\epsilon_a E_o^2}{2u'\alpha_3} \overset{(+)}{-} \sqrt{\frac{\epsilon_a E_o^4}{4u'^2\alpha_3^2} - \frac{|\alpha_2|}{\alpha_3}} \tag{30}$$

It will be shown later (see Eqs. (46) and (50)) that the stable solution of Eq. (29) will correspond to the minus sign in Eq. (30). We immediately see that without the electric field $\tan\theta_n$ will be nonexisting as the argument under the square root sign in Eq. (30) will be negative. Applying a strong enough electric field we can compensate for the negative term under the square root sign thus regaining flow alignment. The lower limit for this stabilizing field will be called the critical electric field and is denoted E_c. Putting the argument under the square root sign equal to zero we obtain

$$E_c^4 = \frac{4u'^2}{\epsilon_a^2}|\alpha_2|\alpha_3 \tag{31}$$

The corresponding flow alignment angle θ_c is given by substituting Eq. (31) into Eq. (30). We shall also bear in mind that ϵ_a can be positive or negative and our formulas in the two cases will differ accordingly.

If $\epsilon_a > 0$ we get (compare the result with Eq. (6))

$$\tan\theta_c = \sqrt{\frac{|\alpha_2|}{\alpha_3}} \tag{32}$$

The angle θ_c lies in the interval $[0, \pi/2]$ and with the expressions for the hydrodynamic and electric torques (Eqs. (2) and (8)) we can understand its existence by looking at Figure 4. With α_3 positive Γ_y^h will be positive for all angles thus tending to rotate the director clockwise everywhere. If $\epsilon_a > 0$ we see that Γ_y^ϵ will be negative if $0 < \theta < \pi/2$ thus tending to rotate the director counterclockwise. For a strong enough field (Figure 4a) we now have two intersections between the graphs of Γ_y^h and $-\Gamma_y^\epsilon$ giving two solutions θ_{n1} and θ_{n2} for the flow alignment angle. As is mentioned earlier we will later show that θ_{n1} is the stable one. From now on we measure the electric field in units of E_c and introduce a real number f (f lies in the interval $[1, \infty]$).

$$f = \frac{E_o}{E_c} \tag{33}$$

Equations (31) and (33) substituted into Eq. (30) then give

$$\tan\theta_n = \sqrt{\frac{|\alpha_2|}{\alpha_3}}\{f^2 \,(\underset{-}{+})\, \sqrt{f^4 - 1}\} = F^+\sqrt{\frac{|\alpha_2|}{\alpha_3}} \tag{34}$$

$$F^+ = f^2 \,(\underset{-}{+})\, \sqrt{f^4 - 1} \tag{35}$$

Decreasing the electric field will make θ_{n1} and θ_{n2} to move closer to each other. In Figure 4b we have drawn the torques when E_o equals E_c. The graphs then touch each other in one point. This angle is of course θ_c. We can also see that θ_c is the largest possible angle for which flow alignment can occur. Studying Figure 4c we see that decreasing the field further, flow alignment will not occur.

If instead $\epsilon_a < 0$ the expression for θ_c will become

$$\tan\theta_c = -\sqrt{\frac{|\alpha_2|}{\alpha_3}} \tag{36}$$

θ_c now lies in the interval $[-\pi/2, 0]$. In Figure 4 $-\Gamma_y^\epsilon$ is now represented by the dotted line. We now have the solution for θ_n for negative θ. With the field defined by Eq. (33) we get

$$\tan\theta_n = \sqrt{\frac{|\alpha_2|}{\alpha_3}}\{-f^2 \,(\underset{-}{+})\, \sqrt{f^4 - 1}\} = F^-\sqrt{\frac{|\alpha_2|}{\alpha_3}} \tag{37}$$

$$F^- = -f^2 \,(\underset{-}{+})\, \sqrt{f^4 - 1} \tag{38}$$

where again the minus sign will correspond to the stable state.

To calculate the boundary layer and the relaxation time we proceed as in Section III. We expand Eq. (10) to linear order in $\delta\theta = \theta - \theta_n$. The resulting equation for $\delta\theta$ is given by Eq. (19) but we will get a new expression for Q which can be shown to be[15]

$$Q = u'\,(|\alpha_2| - \alpha_3)\sin 2\theta_n + \epsilon_a E_o \cos 2\theta_n \tag{39}$$

Using the relations

$$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} \tag{40}$$

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \tag{41}$$

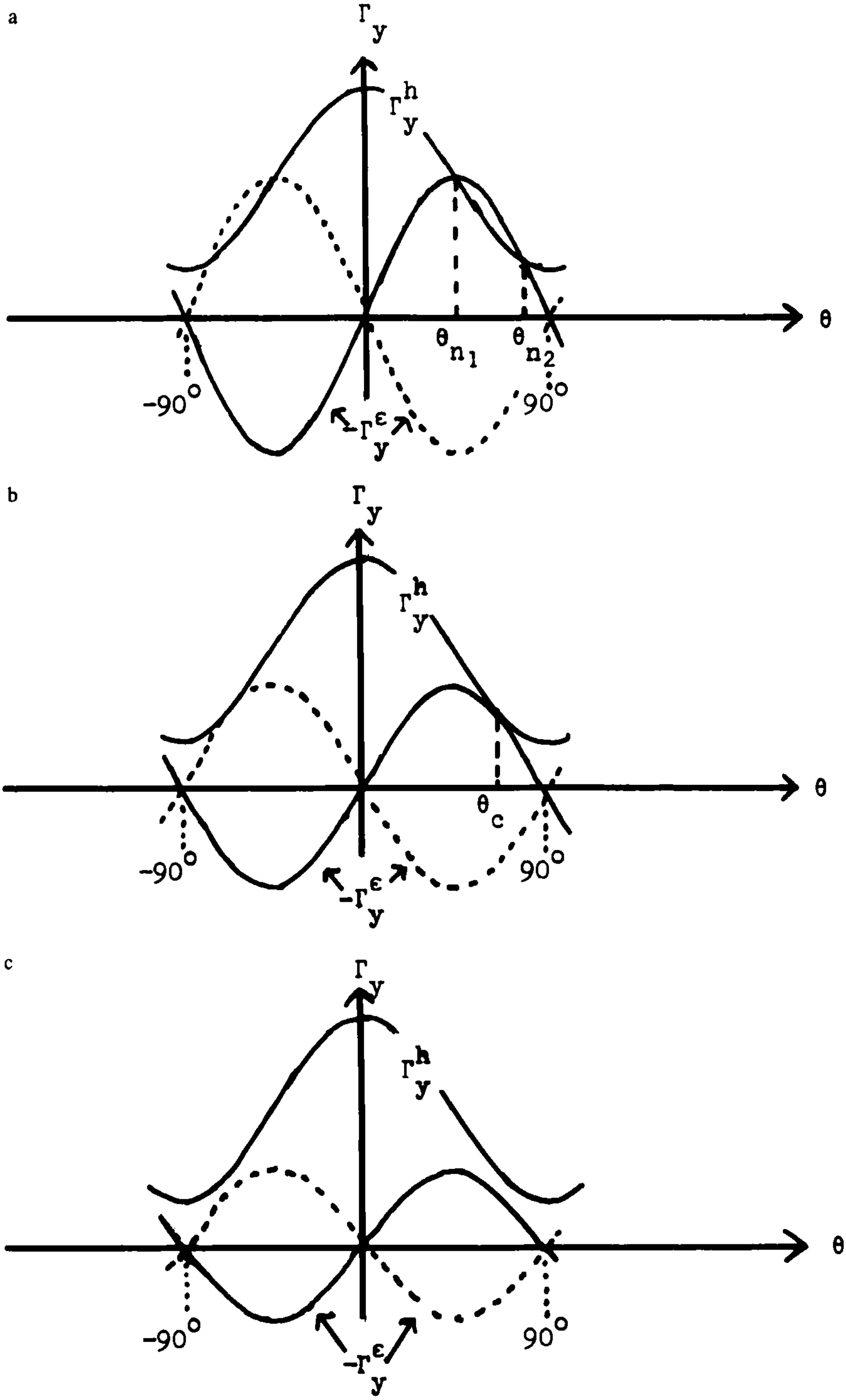

FIGURE 4 Hydrodynamic (Γ_y^h) and electric (Γ_y^ϵ) torques plotted as functions of θ. For $\epsilon_a > 0$ the electric torque is drawn with a full line and for $\epsilon_a < 0$ it is drawn with a dashed line. (a) When $E_o > E_c$ and $\epsilon_a > 0$ we have two possible solutions for the flow alignment angle, θ_{n1} and θ_{n2}. It is shown in Section IV that θ_{n1} is the stable one. E_c is given by Eq. (31). (When $\epsilon_a < 0$ we will have similar solutions for $\theta < 0$) (b) When $E_0 = E_c$ we have only one solution $\theta_c = \arctan \pm \sqrt{|\alpha_2|/\alpha_3}$. The plus sign holds when $\epsilon_a > 0$ and is the one shown in the figure. When $\epsilon_a < 0$ the minus sign is valid. (c) When $E_o < E_c$ flow alignment is not possible.

together with Eq. (34) we can simplify Q. First we introduce ϵ as the ratio between α_3 and $|\alpha_2|$.

$$\epsilon = \alpha_3/|\alpha_2| \tag{42}$$

If $\epsilon_a > 0$ we get

$$Q^+ = Q_o h^+ (f, \epsilon) \tag{43}$$

$$Q_o = 2u'\sqrt{|\alpha_2|\alpha_3} \tag{44}$$

$$h^+ (f, \epsilon) = \frac{F^+ - f^2 F^{+2} + \epsilon(f^2 - F^+)}{\epsilon + F^{+2}} \tag{45}$$

Fortunately using Eq. (35) we can simplify Eq. (45) and prove that $h^+ (f, \epsilon)$ is independent of ϵ.

$$h^+ (f, \epsilon) = f^2 - F^+ = \overset{(-)}{+} \sqrt{f^4 - 1} \tag{46}$$

As can be seen from Eq. (21) or (24) Q must be positive to describe a state which is stable against fluctuations. We then have to choose the lower sign in Eq. (46) which confirms our choice of the minus sign in Eq. (34). We then can write Q^+ as a function of the electric field

$$Q^+ = Q_o\sqrt{f^4 - 1} \tag{47}$$

If instead $\epsilon_a < 0$ we will have to change some signs in our calculations. The result will be

$$Q^- = Q_o h^-(f, \epsilon) \tag{48}$$

$$h^-(f, \epsilon) = \frac{F^- + f^2 F^{-2} - \epsilon(f^2 + F^-)}{\epsilon + F^{-2}} \tag{49}$$

$h^-(f, \epsilon)$ can be simplified

$$h^-(f, \epsilon) = \overset{(-)}{+} \sqrt{f^4 - 1} \tag{50}$$

We again have to choose the lower sign by stability considerations. This confirms our choice of the minus sign in Eq. (37) and we can write

$$Q^- = Q_o\sqrt{f^4 - 1} \tag{51}$$

We see by comparing Eq. (47) and (51) that

$$Q^+ = Q^- = Q \tag{52}$$

Substituting Q into Eqs. (23) and (26) now give expressions for ξ and τ valid for both positive and negative dielectric anisotropy.

$$\xi = \xi_o(f^4 - 1)^{-1/4} \tag{53}$$

$$\xi_o = \sqrt{K/2u'\sqrt{|\alpha_2|\alpha_3}} \tag{54}$$

$$\tau = \tau_o(f^4 - 1)^{-1/2} \tag{55}$$

$$\tau_o = \gamma_1/2u'\sqrt{|\alpha_2|\alpha_3} \tag{56}$$

We now go on discussing the stability of these solutions with respect to fluctuations which take the director out of the shear plane. The hydrodynamic torque acting on the director is such that if $0° < \theta < 90°$ a fluctuation taking the director out of the shear plane will give rise to a torque tending to decrease the fluctuation thus stabilizing the flow.[16] If on the other hand $-90° < \theta < 0°$ a fluctuation will give rise to a destabilizing torque tending to increase the fluctuation. Studying the electric torque we know that for positive ϵ_a the director will tend to be parallel to the field while for negative ϵ_a it will tend to be perpendicular to it. Thus analyzing our solutions with respect to fluctuations out of the shear plane we note that in the case $\epsilon_a > 0$ ($\theta_n \in [0, 90]$) both the hydrodynamic and the electric torque will act stabilizing on the director. On the other hand we see that $\epsilon_a < 0$ ($\theta_n \in [-90, 0]$) both the hydrodynamic and the electric torque will act destabilizing which should lead to an instability for high enough fields and/or shear rates. We will not further on discuss the nature of this instability but merely mention once more that for all known substances with positive α_3 also ϵ_a is positive which assures that all experimental situations based on our equations will be stable with respect to fluctuations which will take the director out of the shear plane.

V RESULTS AND DISCUSSION

The main results of this work are the expressions for the flow alignment angle, the boundary layer and the relaxation time as functions of electric field. The boundary layer and the relaxation time are given by Eqs. (53)–(56). These equations are valid for both positive and negative dielectric anisotropy, but we should observe the possibility of the director going out of the shear plane when $\epsilon_a < 0$. For the flow alignment angle the result differs slightly with the sign of ϵ_a. We can write the result as (the plus sign holds for $\epsilon_a > 0$ and the minus sign for $\epsilon_a < 0$)

$$\tan \theta_n = \sqrt{\frac{|\alpha_2|}{\alpha_3}}\{\pm f^2 - \sqrt{f^4 - 1}\} \tag{57}$$

The applied field is given by

$$f = \frac{E_o}{E_c} \tag{33}$$

$$E_c^4 = \frac{4u'^2}{\epsilon_a^2}|\alpha_2|\alpha_3 \tag{31}$$

Comparing these results with the corresponding expressions for the shear flow when α_3 is negative (Eqs. (6), (23) and (26)) we can see that applying an appropriate electric field produces a flow pattern very similar to this. The only thing we have to change when going from the case $\alpha_3 < 0$ to the case $\alpha_3 > 0$ stabilized by an electric field is that we have to take the absolute value of α_2 in our formulas and correct them with a function of the ratios between the applied electric field and the so called critical electric field. Looking at Figure 5, we can see how θ_n, ξ and τ varies with the applied field. The dashed horizontal lines give the corresponding values for a nematic where α_3 is negative and has the same absolute value as in our case. First we notice that both ξ and τ diverges when the field equals E_c. This can easily be understood because here the electric torque just balances the hydrodynamic one so the effect from the boundary conditions should be able to penetrate very far into the sample. This exact balance should also make the time to reach this state very long. Of course we can not talk of a boundary layer which is infinite, so our fundamental assumption that the flow can be approximated as in Figure 1b will not be valid here. However we see that we do not have to increase the electric field very much to decrease ξ and τ to values reasonable for our approximation. When we have

$$E_o = \sqrt[4]{2}\, E_c \simeq 1,\ 18 E_c \tag{58}$$

we can see that ξ and τ equals the values they would have taken if we had studied a shear flow without electric field and with all parameters unchanged except that α_3 would be negative but with the same absolute value.

Increasing the field further we see that the boundary layer approximation gets even better. Finally substitute Eqs. (31) and (33) into Eqs. (53)–(56) and rewrite ξ and τ as

$$\xi = \sqrt{\frac{K}{|\epsilon_a|}}\,(E_o^4 - E_c^4)^{-1/4} \tag{59}$$

$$\tau = \frac{\gamma_1}{|\epsilon_a|}\,(E_o^4 - E_c^4)^{-1/2} \tag{60}$$

The formulas derived in this work has been used by the authors to measure the temperature dependence of the Leslie viscosities α_2 and α_3 for the nematic liquid crystal 8CB (octyl-cyano-biphenyl).[12] For 8CB ϵ_a is positive so the instability discussed in Section IV will not be of any importance here. We have found that for 8CB $\alpha_3 > 0$ in the whole nematic range, except for a small temperature interval of one degree just below the clearing point. For more detailed information of these measurements the reader is referred to Ref. 12.

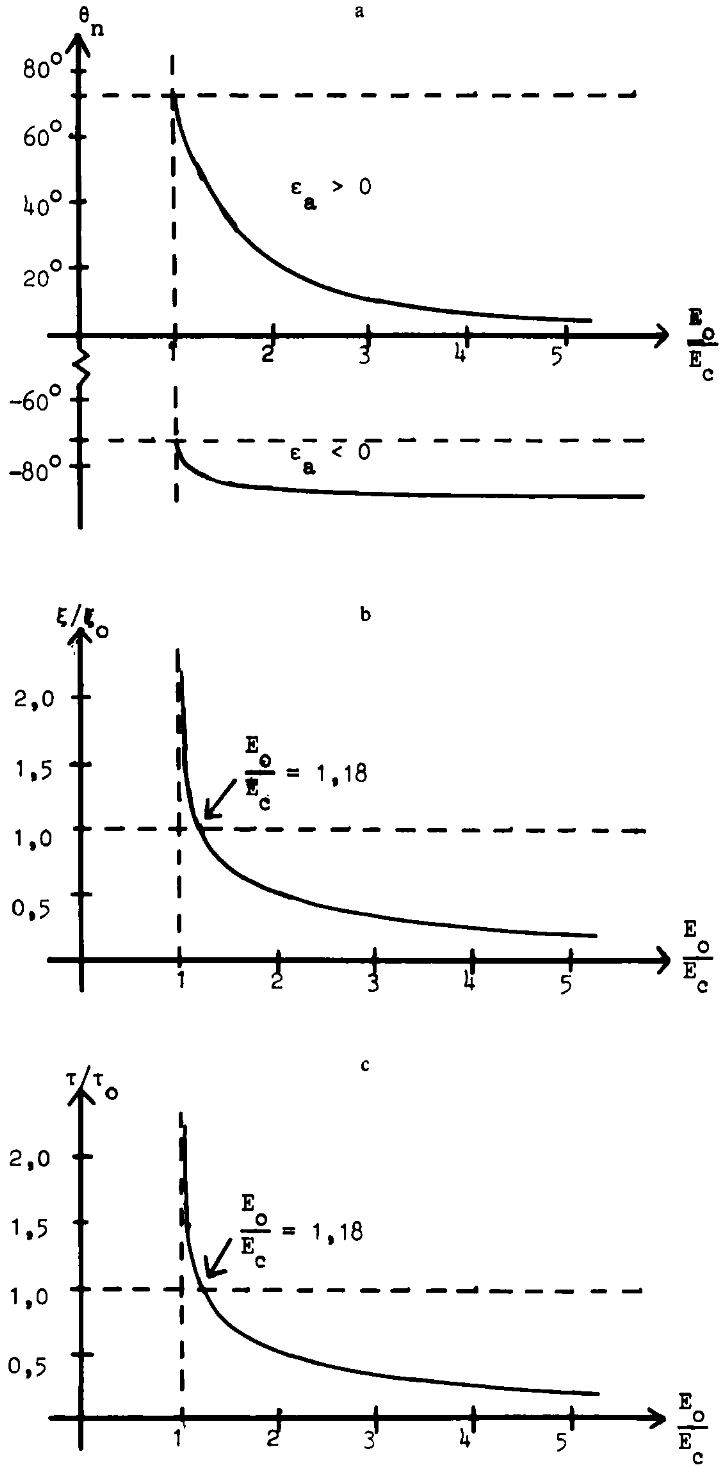

FIGURE 5 Flow alignment angle (a), boundary layer (b) and relaxation time (c) as functions of electric field when $|\alpha_2| = 10\alpha_3$ ($\alpha_3 > 0$). When $E_o \rightarrow E_c$, $\tan\theta_n \rightarrow \pm\sqrt{|\alpha_2|/\alpha_3}$ while ξ and τ diverge.

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